arXiv:2212.12007v1 [cs.CY] 22 Dec 2022

Designing Equitable Transportation Networks (Preprint)

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1 Introduction

Public transit enables people to access employment, healthcare, education, and community resources, which allows members of communities to interact with one another. As a result, this aids the growth and expansion of businesses [1]. Public transit is not only important for individuals; it plays two essential roles in ensuring *justice* to society. First, it distributes social and economic benefits, and second, it links the capabilities of the people, thereby enhancing what people can accomplish as a society [2, 3]. While access to transit infrastructure is vital in general, it is a more critical need for some people than others, i.e., a section of the population depends on public transit for their basic requirements (e.g., access to employment) more so than others [4]. As a result, it is imperative that policymakers design public transit infrastructure equitably, i.e., the design process must explicitly take into account the diverse requirements (and dependencies) of people who use transit. A fundamental problem in strategic transit design is line planning, which seeks to design transit lines and frequencies that serve the given travel demand [5], either at minimum cost or subject to a budget constraint. While the problem has been extensively studied, equity considerations are often absent from traditional line planning literature. This is somewhat counter-intuitive, as it is well-understood that equitable transit is imminently desirable [6, 7, 2, 3]. Moreover, it is not easy to precisely define what *equity* means in this context; for example, Rock et al. [6] present a comprehensive overview of different philosophical trains of thought affecting the design of public transportation and point out that the different notions of equity naturally lead to different outcomes. This paper lays some necessary groundwork to achieve equitable line planning. We introduce a mathematical programming formulation for transit network design with explicit equity considerations (with respect to a measure of *level-of-service*). Importantly, our formulation is *linear*, which is advantageous from an integer programming point of view. We believe our model is a stepping stone to designing equitable transit networks.

2 Models for Equitable Transit Networks

Let $G = (V, A)$ be a strongly-connected directed graph representing the underlying network on which transit has to be installed. Let $|V| = n$ and $|A| = m$. There are lengths $\ell : A \to \mathbb{R}_{\geq 0}$, costs $c : A \to \mathbb{R}_{\geq 0}$, and a budget $B \in \mathbb{R}_{\geq 0}$ which may not be exceeded. In this preliminary version of our work, we assume arcs are uncapacitated. The solution space consists of subsets $A_R \subseteq A$ ensuring each node $u \in V$ has equal in and out degree (so that transit routes are self-rebalancing) and for which $\sum_{a \in A_R} c_a \leq B$.

Let $\mathcal{D} = \{(o,d) \in V \times V : u \neq v\}$. For each origin-destination pair $(o,d) \in \mathcal{D}$, let $b_{od} \in \mathbb{R}_{\geq 0}$ represent the number of people who want to travel from o to d (e.g., in a morning commute pattern). Similarly, let $p_{od} \in (0,1)$ be the priority of serving a passenger traveling from o to d (e.g., need-based). We refer to $\mathbf{b} = (b_{od})_{(o,d)\in\mathcal{D}} \in \mathbb{N}^{n(n-1)}$ and $\mathbf{p} = (p_{od})_{(o,d)\in\mathcal{D}} \in \mathbb{N}^{n(n-1)}$ as the *travel demand profile* and *priority profile*, respectively, and estimate them from data (see Section 3).

A social welfare function reflects aggregate level of service. To this end, we define level of service not as a binary quantity—*given $A_R \subseteq A$ and an origin-destination pair (o,d), is it possible to travel from o to d using only the arcs in A_R?*—but as a "smoother" quantity—*given $A_R \subseteq A$ and an origin-destination pair (o,d), how "good" is the travel from o to d using only the arcs in A_R?* Here, "good" is with respect to level of service offered by the alternative: the use of personal vehicles.

Given $(o,d) \in \mathcal{D}$, let $\ell^*_{od} \in \overline{\mathbb{R}}_{\geq 0}$ denote shortest path distance (with respect to ℓ) from o to d in A. Similarly, let $\ell_{od}(A_R) \in \overline{\mathbb{R}}_{\geq 0}$ denote shortest path distance from o to d in A_R and note that, depending on A_R, it may be that $\ell_{od}(A_R) = \infty$. For example, this holds if $A_R = \emptyset$. In any case, note that $\ell_{od}(A_R) \geq \ell^*_{od}$ for all $A_R \subseteq A$ and $(o,d) \in \mathcal{D}$. Let $\alpha \in \mathbb{R}_{\geq 1}$ be a model parameter reflecting the extent to which passengers tolerate detours; as a multiplicative factor of the shortest path distance ℓ^*_{od}. Given A_R, we assume each passenger who wants to travel from o to d reaps unit *utility* if $\ell_{od}(A_R) = \ell^*_{od}$, zero utility if $\ell_{od}(A_R) \geq \alpha \cdot \ell^*_{od}$, and otherwise reaps utility that interpolates linearly between the points $(\ell^*_{od}, 1)$ and $(\alpha \cdot \ell^*_{od}, 0)$. Formally, for each $(o,d) \in \mathcal{D}$ we have a *utility function* $u_{od} : 2^A \to [0,1]$ where

$$u_{od}(A_R) := \begin{cases} -\frac{\ell_{od}(A_R)}{\ell^*_{od}\cdot(\alpha-1)} + \frac{\alpha}{(\alpha-1)}, & \text{if } \ell^*_{od} \leq \ell_{od}(A_R) < \alpha \cdot \ell^*_{od}, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Given $A_R \subseteq A$, let $\mathbf{u}(A_R) = (u_{od}(A_R))_{(o,d)\in\mathcal{D}} \in [0,1]^{n(n-1)}$ be the *utility profile*. Then, a social welfare function $w : [0,1]^{n(n-1)} \to \mathbb{R}_{\geq 0}$ takes the utility profile $\mathbf{u}(A_R)$ (and implicitly the travel demand profile $\mathbf{b}$ and a priority profile $\mathbf{p}$) to compute some aggregate measure of welfare.

We now present a mixed-integer linear programming model describing the solution space and linearizing Equation (1). We have installation variables $x \in \{0,1\}^m$ indicating the installed network, connectivity variables $y \in \{0,1\}^{n(n-1)}$ indicating the origin-destination pairs to be connected, flow variables $f \in \{0,1\}^{m\times n(n-1)}$ delineating the paths through which said connectivity takes place, length variables $\ell \in \mathbb{R}^{n(n-1)}_{\geq 0}$ recovering the lengths of said paths, and utility variables $u \in \mathbb{R}^{n(n-1)}_{\geq 0}$ quantifying the level of service they offer. We maximize social welfare functions over the region[1]

$$P = \left\{ \begin{array}{ll} x & \in \{0,1\}^m \\ y & \in \{0,1\}^{n(n-1)} \\ f & \in \{0,1\}^{m\times n(n-1)} \\ \ell & \in \mathbb{R}^{n(n-1)}_{\geq 0} \\ u & \in [0,1]^{n(n-1)} \end{array} : \begin{array}{rll} \sum_{a\in A} c_a x_a & \leq B, & \\ \sum_{a\in\delta^+(u)} x_a - \sum_{a\in\delta^-(u)} x_a & = 0, & \forall u \in V \\ \sum_{a\in\delta^+(u)} f^{od}_a - \sum_{a\in\delta^-(u)} f^{od}_a & = y_{od}\cdot\left(\mathbf{1}_{\{u=o\}} - \mathbf{1}_{\{u=d\}}\right), & \forall (o,d)\in\mathcal{D}, u \in V \\ f^{od}_a & \leq x_a, & \forall (o,d)\in\mathcal{D}, a\in A \\ \ell_{od} & = \sum_{a\in A} \ell_a \cdot f^{od}_a, & \forall (o,d)\in\mathcal{D} \\ \ell_{od} & \leq (\alpha\cdot\ell^*_{od})\cdot y_{od}, & \forall (o,d)\in\mathcal{D} \\ u_{od} & = -\frac{\ell_{od}}{\ell^*_{od}\cdot(\alpha-1)} + \frac{\alpha}{(\alpha-1)} - \frac{\alpha}{\alpha-1}\cdot(1-y_{od}), & \forall (o,d)\in\mathcal{D} \end{array} \right\}.$$

The first four sets of constraints defining P correspond to an uncapacitated multi-commodity flow formulation. The constraints $\ell_{od} = \sum_{a\in A} \ell_a \cdot f^{od}_a$ for all (o,d) recover the lengths of the paths through which origin-destination pairs are connected, whereas the constraints $\ell_{od} \leq (\alpha \cdot \ell^*_{od}) \cdot y_{od}$ for all (o,d) ensure said lengths are "tolerable." The last set of constraints implement (1). The reader might observe differences between (1) and how it is implemented in P, and that the constraints $\ell_{od} \leq (\alpha \cdot \ell^*_{od}) \cdot y_{od}$ and $\ell_{od} = \sum_{a\in A} \ell_a \cdot f^{od}_a$ for all (o,d) might lead to the infeasibility (under P) of solutions that are otherwise admissible under the abstract model. This is by design, as we aim to model Equation (1) with only linear constraints. We find that P is a correct formulation.

Theorem 1 *Let $w : [0,1]^{n(n-1)} \to \mathbb{R}_{\geq 0}$ be monotonic increasing. Then, $A_R \subseteq A$ is a solution maximizing $w(\mathbf{u}(A_R))$ if and only if there is $(x,y,f,\ell,u) \in P$ maximizing $w(u)$ with $u = \mathbf{u}(A_R)$.*

For any origin-destination pair $(o,d) \in \mathcal{D}$, we say the *priority-adjusted* utility of a passenger traveling from o to d is $p_{od} \cdot u_{od} \in [0,1)$. Then, if $b_{od} \in \mathbb{N}$ is the number of people who want to travel from o to d, their total

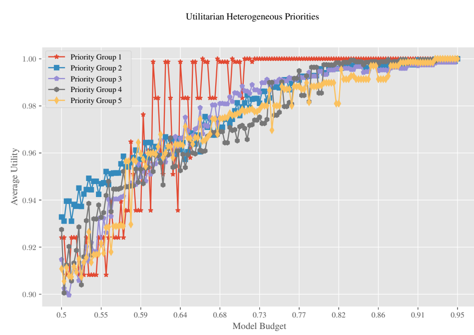
[1]For each $u \in V$, let $\delta^+(u)$ and $\delta^-(u)$ denote the outgoing and incoming arcs of u in G, respectively.

priority-adjusted utility is $b_{od} \cdot (p_{od} \cdot u_{od}) \in [0, b_{od})$. A priority-adjusted utilitarian social welfare function computes the sum of priority-adjusted utilities. Therefore, we define the *maximum priority-adjusted ridership* problem as $\max_{(x,y,f,\ell,u)\in P} \sum_{(o,d)\in\mathcal{D}} b_{od} \cdot (p_{od} \cdot u_{od})$. (This problem can be easily linearized). Similarly, we define the *maximum priority-adjusted coverage* problem as $\max_{(x,y,f,\ell,u)\in P} \min_{(o,d)\in\mathcal{D}} (1 - p_{od}) \cdot u_{od}$. This leads to a family of *maximum priority-adjusted trade-off* problems

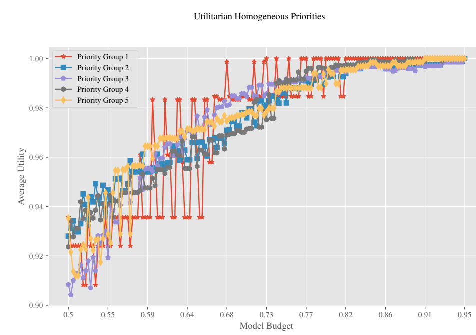
$$\max_{(x,y,f,\ell,u)\in P} \gamma \cdot \sum_{(o,d)\in\mathcal{D}} b_{od} \cdot (p_{od} \cdot u_{od}) + (1-\gamma) \cdot \min_{(o,d)\in\mathcal{D}} (1 - p_{od}) \cdot u_{od}, \tag{2}$$

where $\gamma \in (0, 1]$. In this way, the parameter γ captures the priorities of the transit network planner.

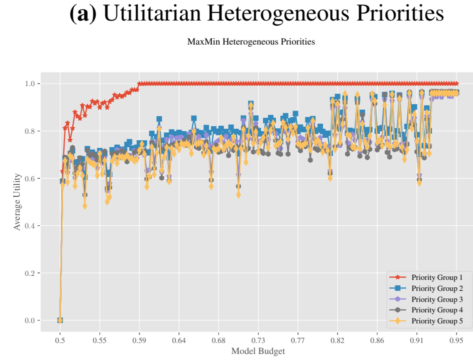
3 Preliminary Experiments and Future Work



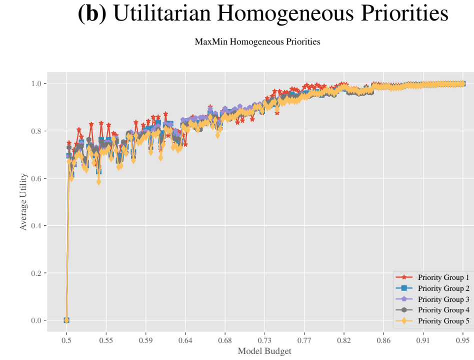
(a) Utilitarian Heterogeneous Priorities



(b) Utilitarian Homogeneous Priorities



(c) MaxMin Heterogeneous Priorities



(d) MaxMin Homogeneous Priorities

Figure 1: Average utility for Utilitarian and MaxMin objectives with $\alpha = 2$, binned into 5 priority groups.

Our implementation uses Julia 1.7.3, JuMP 1.1.1, and Gurobi 9.5.1 [8].

The geographical area under consideration is Chattanooga, Tennessee, Hamilton County. The Census Tracts served by Chattanooga Area Regional Transportation Authority (CARTA) correspond to the number of nodes in the underlying network. The edges represent the shortest major roadway that connects neighboring Census Tracts, each having a cost equivalent to the roadway's length (in kilometers). The origin-destination pairs are from Longitudinal Employer-Household Dynamics Origin-Destination Employment Statistics (LODES 7) [9]. Priority scores are calculated using the origin's average household income and % of private car ownership. These statistics are gathered from Census Bureau Tables, the American Community Survey Data (ACS) [10] [11].

For each priority group, average utility is calculated ex-post to quantify level-of-service. A budget of 1 represents the total cost over all edges of the graph. A budget of 0.95 represents the required minimum budget to serve all pairs by their shortest path. Figures 1a,c show that the average utility for the priority group most in *need* (Group 1), is higher than in Figures 1b,d. This demonstrates the advantage of implementing heterogeneous priorities within the models through the utility function.

Future work will consist of applying our approach to different cities and experimenting with different priority distributions. More realistic models will be derived, in particular enhancing our model with more realistic features that take it from our bare-bones network design formulation toward a full-blown line planning formulation.

Acknowledgement This work is supported by the National Science Foundation under Award Numbers 1952011 and 1818901. The authors acknowledge help from Rishav Sen for generating the OD data for Chattanooga.

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